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MAGNETOPHORETIC POTENTIAL OF A PLANE-ORDERED SYSTEM OF FERROCYLINDERS. 2. RECTANGULAR CYLINDERS

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The magnetophoretic properties of a system of equidistant identical rectangular ferromagnetic cylinders located in one plane and exposed to a uniform magnetic field are considered. The analytical solution of the problem on distribution of the magnetic field and the magnetophoretic potential in magnetization of the system to saturation along the packing plane and across it is obtained. The influence of geometric and magnetic parameters on the magnetophoretic properties of the system is investigated.

Since the mid-1970s, the method of high-gradient magnetic separation has attracted attention in many spheres of activity, including water and gas purification, purification of clays, chemical technologies, medicine, and biology (see [1–8]). High-gradient magnetic filters are created in practice by application of a strong homogeneous magnetic field to a volume in which small ferromagnetic bodies are distributed. The smaller the separated particles and the weaker their magnetic properties, the stronger must be the magnetic field and the smaller the size of the elements of the ferromagnetic packing. The efficiency of a magnetic filter is determined by a number of factors, including the shape of the ferromagnetic bodies and the geometry of their packing, the direction of magnetization of the packing and the direction of suspension flow, and the character of the magnetic properties of the particles separated. Information on the distribution of the magnetophoretic potential in different ferromagnetic structures is required for the creation of efficient filters of specific application. Of greatest interest from both the theoretical and practical viewpoints are ordered (periodic) structures. The periodicity of the structure simplifies the analysis and enables one to realize the principle of accumulation of the effect of magnetophoretic separation in motion of the suspension, as evidenced by the results of investigation of a linearly ordered system of ferromagnetic spheres [9] and a plane-ordered system of circular ferrocylinders [10]. In the present work, we have studied the magnetophoretic potential of a structure representing a plane-ordered system of ferromagnetic cylinders of a rectangular cross section.

Magnetic Self-Field of the Magnetized Structure. Let the cylinders lie in the plane (X, Y) along the X axis of the Cartesian coordinate system X, Y, Z (see Fig. 1). The origin of the coordinate system is located on the axis of one cylinder to which the number $n = 0$ is applied. The cylinder width is $2a$, the height is $2B$, and the step of packing is S. The direction of the X, Y, and Z axes is specified by the unit vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$. The external magnetic field $\mathbf{H}_0 = H_0\mathbf{e}$ is directed along ($\mathbf{e} = \mathbf{j}$) or across ($\mathbf{e} = \mathbf{k}$) the packing plane. We assume that the field is large ($H_0 \succ 4\pi M_s$), so that the cylinders are magnetized to saturation. Let $A_0^{(0)}(X_0, Y_0, Z_0)$ be the point belonging to the cylinder with No. 0. Each of such points generates a set of points $A_0^{(n)}(X_0, Y_0^{(n)}, Z_0)$, where $Y_0^{(n)} = Y_0 + nS$, belonging to all the remaining cylinders.

The intensity of the field produced at the point $A(X, Y, Z)$ of an infinitely small volume $dX_0dY_0^{(n)}dZ_0$ of the cylinder n, which is constructed at the point $A_0^{(n)}$, is determined by the relation

$$d\mathbf{H}'\,(\mathrm{A}_0^{(n)}, \mathrm{A}) = -\,M_s\mathbf{K}\,(\mathrm{A}_0^{(n)}, \mathrm{A})\; dx_0dy_0dz_0\,, \tag{1}$$

where

$$\mathbf{K}\,(\mathrm{A}_0^{(n)}, \mathrm{A}) = \frac{1}{r^3_{\mathrm{A}_0^{(n)}\mathrm{A}}}\left[\mathbf{e} - 3\,\frac{(\mathbf{e}\mathbf{r}_{\mathrm{A}_0^{(n)}\mathrm{A}})\,\mathbf{r}_{\mathrm{A}_0^{(n)}\mathrm{A}}}{\mathbf{r}^2_{\mathrm{A}_0^{(n)}\mathrm{A}}}\right], \quad \mathbf{r}_{\mathrm{A}_0^{(n)}\mathrm{A}} = (x - x_0)\,\mathbf{i} + (y - y_0 - ns)\,\mathbf{j} + (z - z_0)\,\mathbf{k}\,.$$

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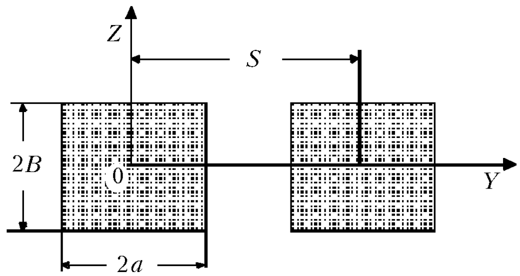

Fig. 1. Geometry of the problem.

In these formulas and in what follows, the distances are measured in the units of the cylinder half-width ($x = X/a$, $b = B/a$, $s = S/a$).

The total field of the structure is computed as follows. First, by integration of relation (1) with respect to x_0 and z_0 we find the field of the thin layer y_0, $y_0 + dy_0$ of the nth cylinder and then carry out summation over n from $-\infty$ to $+\infty$ and integration of the result with respect to y_0, i.e.,

$$\mathbf{H}'(\mathrm{A}) = -M_s \int_{-1}^{1} \left\{ \sum_{n=-\infty}^{\infty} \int_{-b}^{b} \int_{-\infty}^{\infty} \mathbf{K}(\mathrm{A}_0^{(n)}, \mathrm{A})\, dx_0 dz_0 \right\} dy_0 . \quad (2)$$

For compact representation of the results we introduce the notation $\hat{y} = y/s$, $\hat{z} = z/s$, $\hat{b} = b/s$, $A(\hat{y}) = \cot(\pi\hat{y})$, $B(\hat{z}, \hat{b}) = \coth[\pi(\hat{z} + \hat{b})]$, and $G(\hat{y}, \hat{z}, \hat{b}) = \arctan[A(s^{-1} + \hat{y})B^{-1}(\hat{z}, \hat{b})]$:

$$\times \ln \frac{\left[B^2(\hat{z}, -\hat{b}) + \left(\dfrac{1 - A(s^{-1})A(\hat{y})}{A(\hat{y}) + A(s^{-1})}\right)^2\right]\left[B^2(\hat{z}, -\hat{b}) + \left(\dfrac{1 + A(s^{-1})A(\hat{y})}{A(\hat{y}) - A(s^{-1})}\right)^2\right]}{\left[B^2(\hat{z}, \hat{b}) + \left(\dfrac{1 + A(s^{-1})A(\hat{y})}{A(\hat{y}) - A(s^{-1})}\right)^2\right]\left[B^2(\hat{z}, \hat{b}) + \left(\dfrac{1 - A(s^{-1})A(\hat{y})}{A(\hat{y}) + A(s^{-1})}\right)^2\right]}, \quad (3)$$

$$N(y, z, b, s) = \frac{1}{\pi}[G(-\hat{y}, \hat{z}, -\hat{b}) + G(\hat{y}, \hat{z}, -\hat{b}) - G(-\hat{y}, \hat{z}, \hat{b}) - G(\hat{y}, \hat{z}, \hat{b})] .$$

The intensity of the self-field of the structure is represented in the form $\mathbf{H}' = 2\pi M_s \mathbf{h}$. Now the results of computation from formula (2) for the cases of longitudinal and transverse magnetization have the form

$$h_y^{\parallel}(y, z, b, s) = N(y, z, b, s), \quad h_z^{\parallel}(y, z, b, s) = T(y, z, b, s), \quad \mathbf{e} = \mathbf{j};$$
$$h_y^{\perp}(y, z, b, s) = T(y, z, b, s), \quad h_z^{\perp}(y, z, b, s) = -N(y, z, b, s), \quad \mathbf{e} = \mathbf{k}. \quad (4)$$

Magnetophoretic Potential of the Magnetized Structure. The total magnetic field of the system is equal to the sum of the magnetizing field and the self-field, $\mathbf{H} = \mathbf{H}_0 + \mathbf{H}'$. According to [9, 10], the force acting on a dia- or paramagnetic particle in a nonuniform magnetic field (on condition that the nonuniformity scale is large as compared to the particle size) can be expressed by the magnetophoretic potential of the filtering structure Φ:

$$\mathbf{F}_m = -\nabla\Phi, \quad \Phi = -\frac{1}{2}\Delta\chi v (2\pi M_s)^2 [h^2 + P\mathbf{eh}], \quad P = H_0/\pi M_s . \quad (5)$$

Here v is the particle volume and $\Delta\chi = \chi - \chi_0$ is the difference of the magnetic susceptibilities of the particle and the carrier liquid. We introduce the dimensionless magnetophoteric potential φ, taking the value of the potential of the magnetic self-field of a circular cylinder at its forward point as the scale [10]:

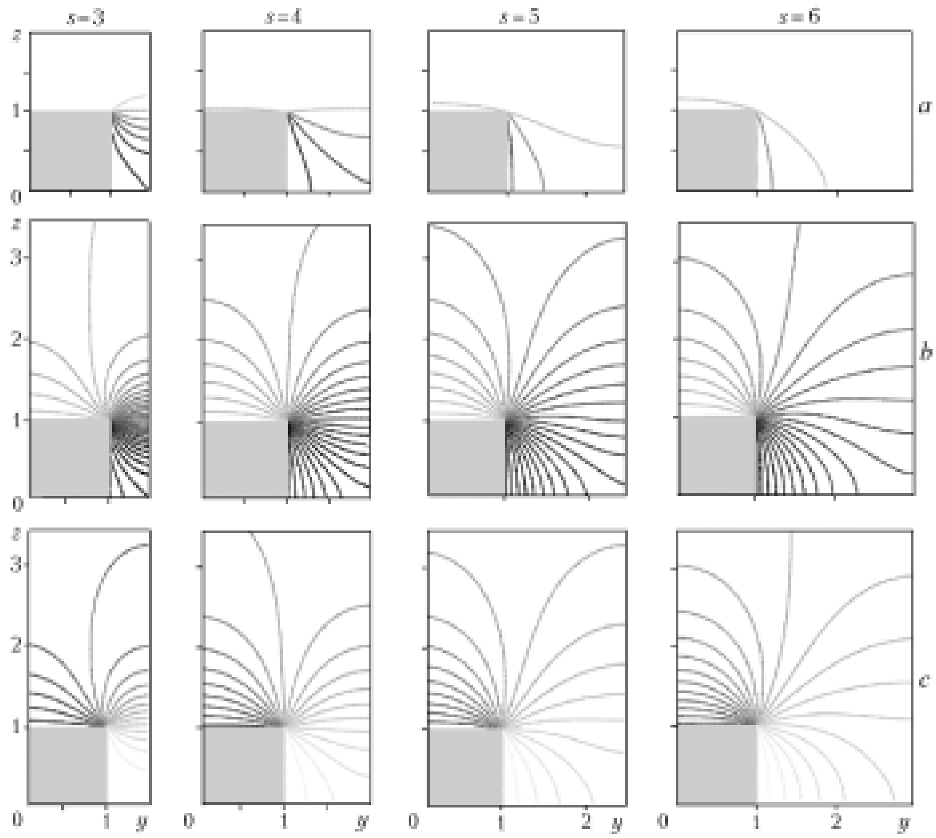

Fig. 2. Isolines of magnetophoretic potential of the system of square ferro-cylinders for different structural steps s and different methods of magnetization: constant magnets in the zero external field (a); soft magnetic cylinders are magnetized by the field P = 4 longitudinally (b) and transversely (c).

$$\Phi^* = \frac{1}{2}\,\Delta\chi v\,(2\pi M_s)^2\,.$$

We have

$$\varphi \equiv \Phi/\Phi^* = \varphi_1 + \varphi_2\,, \quad \varphi_1 = -\,\mathbf{h}^2\,, \quad \varphi_2 = -\,P\,(\mathbf{eh})\,. \tag{6}$$

Using relations (10) we find for the longitudinally and transversely magnetized structures

$$\varphi_1^{\parallel} = \varphi_1^{\perp} = -\,N^2 - T^2\,, \quad \varphi_2^{\parallel} = -\,\varphi_2^{\perp} = -\,PN\,. \tag{7}$$

The minimum value of P is determined by the condition of magnetization of the cylinders to saturation. The necessary field is evaluated by the relation $H_0 \geq 4\pi M_s$. Consequently, $P \geq 4$.

We note (see [9, 10]) that the magnetophoretic potential made dimensionless by the method adopted refers to paramagnetic particles ($\Delta\chi > 0$) and that taken with an opposite sign refers to diamagnetic particles ($\Delta\chi < 0$). In other words, paramagnetic particles move in the direction of the minimum potential φ whereas diamagnetic particles move in the direction of the maximum potential. The quantity φ_1 expresses the magnetophoretic potential of the self-field of a filtering structure. In particular, if the cylinders are manufactured from a hard magnetic material with remanence M_s, the quantity φ_1 comprises the total potential of the system with a switched-off external field. The quantity φ_2 represents a result of the interference of the self-field and the external field. There are significant qualitative differences between the intrinsic potential and the interference potential. The first is strictly negative and consequently has the at-

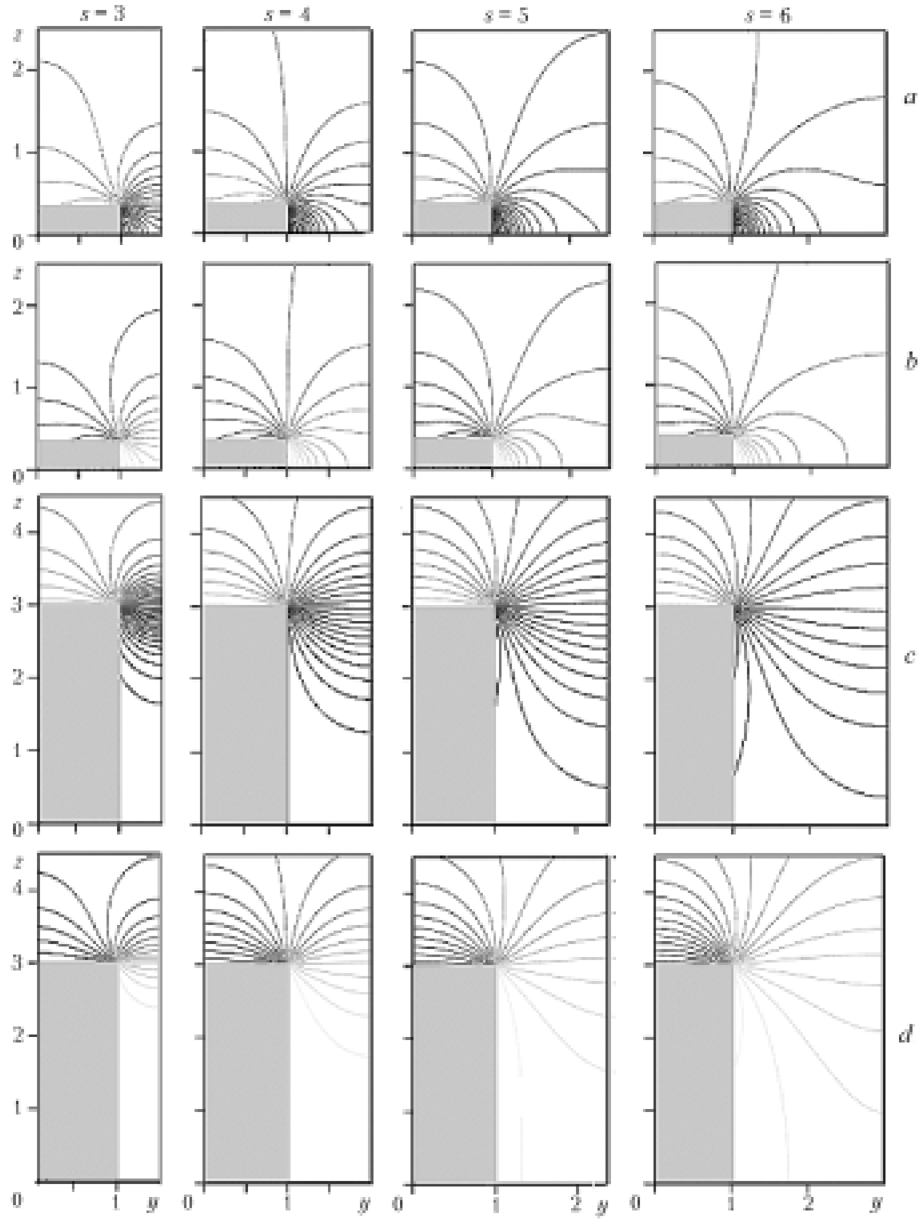

Fig. 3. Isolines of magnetophoretic potential of the system of wide (b = 1/3 (a, b)) and narrow (b = 3 (c, d)) ferrocylinders with different structural steps s for the cases of longitudinal (a, c) and transverse (b, d) magnetization (P = 4).

traction of paramagnetic particles and the repulsion of diamagnetic particles as its effect. The second can change its sign at different points, thus creating prerequisites for sedimentation of diamagnetic particles. The value of the intrinsic potential of the structure (naturally, on attainment of magnetic saturation) is independent of the external-field intensity, whereas the interference potential increases in proportion to H_0.

Thus, the distribution of magnetophoretic potential is determined by the direction of magnetization, the dimensionless field intensity P, the ratio of the height of the cylinder to its width b, and the ratio of the structural step to the cylinder half-width. To visualize the structure and intensity of the magnetophoretic field we consider the pattern of isolines of the potential. In the case of square cylinders (b = 1) it is shown in Fig. 2 for different structural steps and different methods of magnetization of the structure. One-fourth of a cylinder is given because of the periodicity of the potential in the y axis and the symmetry relative to the plane z = 0. The color of the isolines (see Fig. 2) becomes more intense with decrease in the corresponding algebraic value of the potential, i.e., paramagnetic particles move in

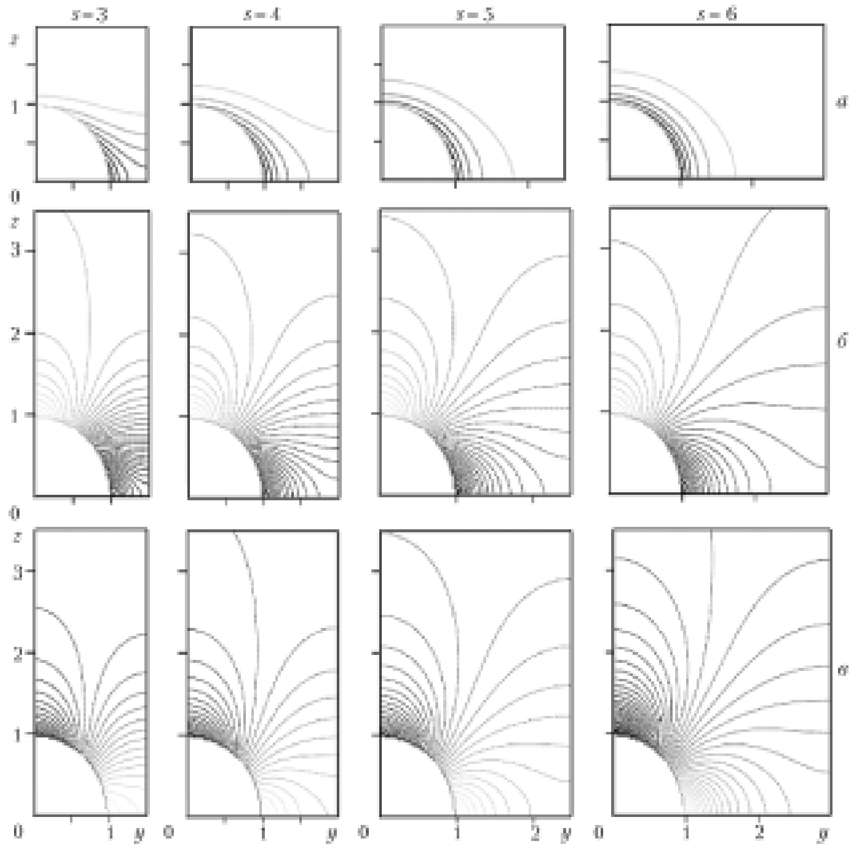

Fig. 4. Isolines of magnetophoretic potential of the system of circular ferro-cylinders under conditions analogous to those in Fig. 2.

the direction of the maximum color, whereas diamagnetic particles move in the direction of the minimum. The number of isolines in each case is equal to the rounded-off value of the quantity $4(\varphi_{max} - \varphi_{min})$, where φ_{max} and φ_{min} are the maximum and minimum values of the potential in the calculation region (on the 200×200 grid). Consequently, the potential difference between neighboring isolines for all the variants is nearly the same ($\Delta\varphi \approx 0.25$) and the density of the isolines enables us to compare the value of the magnetophoretic force $-\nabla\varphi$ in different situations. Figure 2 shows the potentials of the longitudinally and transversely magnetized (in the field $P = 4$) system together with its intrinsic potential in a zero field.

According to the pattern presented, the structure of the magnetophoretic field is determined by a set of the singular points at the corners of the cylinders. All the isolines pass through the corners in the vicinity of which the magnetophoretic force attains its maximum value. One part of the isolines leaving a given corner of a cylinder enters the adjacent corner of the neighboring cylinder and the other part enters the adjacent corners of the same cylinder. The bundles of isolines connecting the neighboring cylinders characterize the collective nature of the system's magnetophoretic field. By convention the potential in the bundles connecting the neighboring cylinders is defined as the collective potential and the potential in the bundles connecting the corners of one and the same cylinder is defined as the individual potential. The first determines the effect of pulling (pushing) of particles deep into the gap between the cylinders, while the second determines the attraction (repulsion) of particles to the cylinders' surface. If the gap between the cylinders is small, most of the potential is concentrated in collective bundles and the pulling (pushing) of particles into the gap is the dominating feature of the magnetophoretic field. As the gap becomes wider, an ever increasing number of isolines goes from the collective bundle to individual bundles, and the effect of pulling of particles into the gap becomes weaker, while the effect of attraction to the cylinder surface is enhanced; the range of action of the magnetophoretic force in the external region is also extended ($|z| > b$).

The influence of the magnetization method is as follows. In the intrinsic-potential field, paramagnetic particles are pulled deep into the gap and are attracted to the entire surface of the cylinder. In the longitudinally magnetized system, there is an analogous but much more intense magnetophoretic interaction in the region of the gap; the direction of the magnetophoretic force changes its sign on the exterior cylinder surface, here diamagnetic particles are attracted. In the transversely magnetized system, the direction of the magnetophoretic force is opposite as compared to that in the longitudinally magnetized system. The intensity of magnetophoretic interaction on the exterior cylinder surface is nearly the same in both cases but it is markedly lower in the gap region in the transversely magnetized system.

Let us consider the influence of the cylinder shape (height-to-width ratio) on the structure of the magnetophoretic field. Figure 3 gives the pattern of the potential isoline for wide (b = 1/3) and narrow (b = 3) cylinders in longitudinal and transverse magnetization (P = 4). The data presented together with the data in Fig. 2 provide a clear answer to the question posed and do not require additional discussion.

It is of interest to compare the magnetophoretic fields of square and circular cylinders. Let us use the results of [10] and construct the field of the latter (see Fig. 4) under conditions analogous to those adopted in Fig. 2 (the same values of the structural step and the field intensity). As we see, apart from the general similarity, the fields of the square and circular cylinders have a number of significant differences.

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NOTATION

S, step of packing of ferrocylinders, cm; a, cylinder half-width, cm; B, cylinder half-height, cm; b, dimensionless half-height of the cylinder; s, dimensionless step of packing; $\mathbf{H}_0$ and $\mathbf{H}'$, intensity of the external magnetic field and the self-field, Oe; $\mathbf{e}$, unit vector in the direction of the external magnetic field; M_s, saturation magnetization, G; P, dimensionless intensity of the field; $\mathbf{h}$, dimensionless self-field of the structure; $h_y^{\parallel}$, $h_y^{\perp}$, $h_z^{\parallel}$, and $h_z^{\perp}$, components of the dimensionless self-field of the structure in longitudinal and transverse magnetization; n, cylinder No.; X, Y, Z, Cartesian coordinates; $A(X, Y, Z)$, arbitrary point; $A_0^{(0)}(X_0, Y_0, Z_0)$, point belonging to the zero cylinder; $A_0^{(n)}$, point belonging to the nth cylinder; x, y, z, dimensionless coordinates (in the units of a); x_0, y_0, z_0, dimensionless coordinates of the points of the zero cylinder; $\hat{x}$, $\hat{y}$, $\hat{z}$, dimensionless coordinates measured by the structural step; $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$, unit vectors of the Cartesian coordinate system; $\mathbf{r}_{A_0^{(n)}A}$, dimensionless radius vector from point $A_0^{(n)}$ to point A; γ, magnetic susceptibility of the separated particles; χ_0, magnetic susceptibility of the carrier liquid; $\Delta\chi = \chi - \chi_0$; Φ, magnetophoretic potential, $g{\cdot}cm^2{\cdot}sec^{-2}$; $\mathbf{F}_m$, magnetophoretic force, $g{\cdot}cm{\cdot}sec^{-2}$; φ, dimensionless magnetophoretic potential; φ_1 and φ_2, components of the dimensionless magnetophoretic potential; A, B, G, T, and N, functions determined by relations (3). Subscripts: s, saturation; m, magnetic; max, maximum; min, minimum.

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